

Exercise 1 Solve the following equations:

$$\begin{array}{llll}
 \text{(a)} x'' - 2x = 0, & \text{(b)} 4x'' - x = 0, & \text{(c)} x'' + 4x' = 0, & \text{(d)} 3x'' - 2x = 0, \\
 \text{(e)} x'' + x' - 2x = 0, & \text{(f)} 2x'' - 10x' + 12x = 0, & \text{(g)} 6x'' - x' - x = 0, & \text{(h)} 3x'' - 2x' - x = 0, \\
 \text{(i)} x'' - 2x' + x = 0, & \text{(j)} 2x'' + 3x' - 2x = 0, & \text{(k)} x'' + x' - 2x = 0, & \text{(l)} 4x'' + 12x' + 9x = 0, \\
 \text{(m)} 2x'' = 0, & \text{(n)} 4x'' - 4x' + x = 0, & \text{(o)} x'' + x = 0, & \text{(p)} x'' + 3x = 0, \\
 \text{(r)} x'' - 6x' + 25x = 0, & \text{(s)} x'' + 6x' + 18x = 0, & \text{(t)} 36x'' - 36x' + 13x = 0, & \text{(u)} x'' - 4x' + 13x = 0.
 \end{array}$$

Answers.

$$\begin{array}{lll}
 \text{(a)} x(t) = C_1 e^{\sqrt{2}t} + C_2 e^{-\sqrt{2}t}, & \text{(b)} x(t) = C_1 e^{t/2} + C_2 e^{-t/2}, & \text{(c)} x(t) = C_1 e^{-4t} + C_2, \\
 \text{(d)} x(t) = C_1 e^{t\sqrt{2/3}} + C_2 e^{-t\sqrt{2/3}}, & \text{(e)} x(t) = C_1 e^{-2t} + C_2 e^t, & \text{(f)} x(t) = C_1 e^{2t} + C_2 e^{3t}, \\
 \text{(g)} x(t) = C_1 e^{-t/3} + C_2 e^{t/2}, & \text{(h)} x(t) = C_1 e^{-t/3} + C_2 e^t, & \text{(i)} x(t) = C_1 e^t + C_2 e^{t^2}, \\
 \text{(j)} x(t) = C_1 e^{t/2} + C_2 e^{-2t}, & \text{(k)} x(t) = C_1 e^{-2t} + C_2 e^t, & \text{(l)} x(t) = C_1 e^{-3t/2} + C_2 e^{-3t/2t}, \\
 \text{(m)} x(t) = C_1 + C_2 t, & \text{(n)} x(t) = C_1 e^{t/2} + C_2 e^{t/2t}, & \text{(o)} x(t) = C_1 \sin t + C_2 \cos t, \\
 \text{(p)} x(t) = C_1 \sin(\sqrt{3}t) + C_2 \cos(\sqrt{3}t), & \text{(r)} x(t) = C_1 e^{3t} \sin(4t) + C_2 e^{3t} \cos(4t), & \text{(s)} x(t) = C_1 e^{-3t} \sin(3t) + C_2 e^{-3t} \cos(3t), \\
 \text{(t)} x(t) = C_1 e^{t/2} \sin \frac{t}{3} + C_2 e^{t/2} \cos \frac{t}{3}, & \text{(u)} x(t) = C_1 e^{2t} \sin(3t) + C_2 e^{2t} \cos(3t).
 \end{array}$$

Exercise 2 Solve the following equations:

$$\begin{array}{ll}
 \text{(a)} x'' - 4x' + 3x = 0, x(0) = 7, x'(0) = 16, & \text{(b)} 2x'' + 4x' - 6x = 0, x(0) = 4, x'(0) = 0, \\
 \text{(c)} x'' + 6x' + 8x = 0, x(0) = 3, x'(0) = 10, & \text{(d)} x'' - x' - 6x = 0, x(0) = 2, x'(0) = 1, \\
 \text{(e)} 2x'' = 0, x(0) = -1, x'(0) = 1, & \text{(f)} x'' + 6x' + 9x = 0, x(0) = 3, x'(0) = -10, \\
 \text{(g)} 4x'' + 4x' + x = 0, x(0) = 1, x'(0) = 0, & \text{(h)} x'' + 100x = 0, x(0) = 1, x'(0) = 10, \\
 \text{(i)} x'' + 4x' + 20x = 0, x(0) = 2, x'(0) = 0, & \text{(j)} x'' - 4x' + 5x = 0, x(0) = -1, x'(0) = 2, \\
 \text{(k)} x'' + 2x' + 5x = 0, x(0) = -5, x'(0) = 3, & \text{(l)} 4x'' - 4x' + 5x = 0, x(0) = 0, x'(0) = -1.
 \end{array}$$

Answers.

$$\begin{array}{lll}
 \text{(a)} x(t) = \frac{1}{2} e^t (9e^{2t} + 5), & \text{(b)} x(t) = e^{-3t} + 3e^t, & \text{(c)} x(t) = e^{(-4t)} (11e^{2t} - 8), \\
 \text{(d)} x(t) = e^{-2t} + e^{3t}, & \text{(e)} x(t) = t - 1, & \text{(f)} x(t) = -e^{-3t} (t - 3), \\
 \text{(g)} x(t) = \frac{1}{2} e^{-t/2} (t + 2), & \text{(h)} x(t) = \sin(10t) + \cos(10t), & \text{(i)} x(t) = e^{-2t} (\sin(4t) + 2 \cos(4t)), \\
 \text{(j)} x(t) = -e^{2t} (\cos t - 4 \sin t), & \text{(k)} x(t) = -e^{-t} (\sin(2t) + 5 \cos(2t)), & \text{(l)} x(t) = -e^{t/2} \sin(t).
 \end{array}$$

Exercise 3

Solve the equations:

$$\begin{array}{lll}
 \text{(a)} x'' + x = t^3, & \text{(b)} x^{(3)} - 3x'' + 3x' - x = e^t, & \text{(c)} 2x^{(3)} + x' = e^t, \\
 \text{(d)} x^{(3)} - 4x'' + 4x' = e^t, & \text{(f)} x^{(3)} - 2x'' - x' + 2x = 2t^2 - 6t + 4, & \\
 \text{(g)} x'' + 3x' - 4x = -13 \sin t, & \text{(h)} x^{(3)} + 3x'' + 3x' + x = 16e^t + t + 3, & \text{(i)} x^{(3)} + 2x'' - x' - 2x = 1 - 4t^3.
 \end{array}$$

Answers.

$$\begin{array}{ll}
 \text{(a)} x(t) = -6t + t^3 + C_1 \cos t + C_2 \sin t, & \text{(b)} x(t) = \frac{t^3 e^t}{6} + C_1 e^t + C_2 t e^t + C_3 t^2 e^t, \\
 \text{(c)} x(t) = \frac{e^t}{3} + C_1 - C_2 \cos \frac{t}{\sqrt{2}} + C_3 \sin \frac{t}{\sqrt{2}}, & \text{(d)} x(t) = e^t + e^{2t} (C_1 + C_2 t) + C_3, \\
 \text{(f)} x(t) = 3 - 2t + t^2 + e^{-t} C_1 + e^t C_2 + e^{2t} C_3, & \text{(g)} x(t) = C_1 e^{-4t} + C_2 e^t + \frac{13}{34} (3 \cos t + 5 \sin t), \\
 \text{(h)} x(t) = 2e^t + t + C_1 e^{-t} + C_2 t e^{-t} + C_3 t^2 e^{-t}, & \text{(i)} x(t) = -8 + 15t - 3t^2 + 2t^3 + C_1 e^{-2t} + C_2 e^{-t} + C_3 e^t.
 \end{array}$$

Exercise 4

Solve the equations:

$$\begin{array}{l}
 \text{(a)} x' + x = a \sin t, x(0) = 0, \\
 \text{(b)} x'' + 4x = \sin t, x(0) = 1, x'(0) = 1, \\
 \text{(c)} x' - 4x'' + 3x = \sin t, x(0) = \frac{11}{5}, x'(0) = \frac{41}{10}.
 \end{array}$$

Answers.

$$\text{(a)} x(t) = \frac{1}{2} a (\sin t - \cos t + e^{-t}), \quad \text{(b)} x(t) = \cos 2t + \frac{1}{3} (\sin t + \sin 2t), \quad \text{(c)} x(t) = e^t + e^{3t} + \frac{1}{10} (2 \cos t + \sin t).$$