A solution to $(1-t^2)x'' - 2tx' + 2x = 0$ is $x = c_1 t + c_2\left(\frac{t}{2}\ln\frac{1+t}{1-t} - 1\right)$.

$$x = c_1 t + c_2\left(\frac{t}{2}\ln\frac{1+t}{1-t} - 1\right)$$

$$x' = c_1 + c_2\left(\frac{1}{2}\ln\frac{1+t}{1-t} + \frac{t}{2}\cdot\frac{1-t}{1+t}\cdot\frac{1\cdot(1-t)-(1+t)\cdot(-1)}{(1-t)^2}\right)$$

$$= c_1 + c_2\left(\frac{1}{2}\ln\frac{1+t}{1-t} + \frac{t}{1-t^2}\right)$$

$$x'' = c_2\left(\frac{1}{2}\cdot\frac{2}{1-t^2} + \frac{1\cdot(1-t^2)-t\cdot(-2t)}{(1-t^2)^2}\right) = \frac{2c_2}{(1-t^2)^2}$$

$$(1-t^2)x'' - 2tx' + 2x = (1-t^2)\cdot\frac{2}{(1-t^2)^2} - 2t\cdot\left[c_1 + c_2\left(\frac{1}{2}\ln\frac{1+t}{1-t} + \frac{t}{1-t^2}\right)\right] + 2\left[c_1 t + c_2\left(\frac{t}{2}\ln\frac{1+t}{1-t} - 1\right)\right]$$

$$= \frac{2c_2}{1-t^2} - 2c_1 t - c_2 t\ln\frac{1+t}{1-t} - \frac{2c_2 t^2}{1-t^2} + 2c_1 t + c_2 t\ln\frac{1+t}{1-t} - 2c_2 = \frac{2c_2}{1-t^2}\left[1 - t^2 - (1-t^2)\right] = 0$$