Exercise Check that the following ODEs and initial value problems are solvable by the given functions.

(1) $y' + y = \sin x$, $y = c_1 e^{-x} + \frac{1}{2}(\sin x - \cos x)$

(2) $y' + y = \sin x$, $y(0) = 0$, $y = \frac{1}{2}(e^{-x} + \sin x - \cos x)$

(3) $(1 - t^2)x'' - 2tx' + 2x = 0$, $x = c_1 t + c_2 \left(\frac{t}{2} \ln \frac{1+t}{1-t} - 1 \right)$

(4) $x''' - x'' - 8x' + 12x = 0$, $x = c_1 e^{-3t} + c_2 e^{2t} + c_3 t e^{2t}$

(5) $x''' - x'' - 8x' + 12x = 104 \sin 2t - 12t + 32$,
$x = c_1 e^{-3t} + c_2 e^{2t} + c_3 t e^{2t} + 3 \cos 2t + 2 \sin 2t$

(6) $x'' - 4x' + 4x = 0$, $x = c_1 e^{2t} + c_2 t e^{2t}$

(7) $x'' - 4x' + 4x = e^{2t}$, $x = \frac{1}{2} t^2 e^{2t} + c_1 e^{2t} + c_2 t e^{2t}$