

**Exercise 1** Find the derivatives of the following functions:

1.  $f(x) = (4x^2 + x)^2$ ,
2.  $f(x) = (1 + \sqrt[3]{x})^2$ ,
3.  $f(x) = \frac{1}{10x^5}$ ,
4.  $f(x) = \frac{x}{2x-1}$ ,
5.  $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ ,
6.  $f(x) = \sqrt{(2x+1)^2 + (3x+1)^2}$ ,
7.  $f(x) = e^{x^2}$ ,
8.  $f(x) = e^{\sin x}$ ,
9.  $f(x) = e^{\sqrt{\sin x}}$ ,
10.  $f(x) = e^{\frac{1}{\sin x}}$ ,
11.  $f(x) = \sin \sqrt{1+x^2}$ ,
12.  $f(x) = x\sqrt{1+x^2}$ ,
13.  $f(x) = \operatorname{ctg}^3 \frac{x}{3}$ ,
14.  $f(x) = \sin^3 x + \cos^3 x$
15.  $f(x) = \sin^3 x^2$ ,
16.  $f(x) = \sqrt{\operatorname{arc} \sin x} - (\operatorname{arc} \operatorname{tg} x)^3$ ,
17.  $f(x) = \operatorname{arc} \sin \frac{1}{x^2} + \ln^5 x$ ,
18.  $f(x) = \frac{1}{3\sqrt{x}} + \frac{x}{x^2+1}$ ,
19.  $f(x) = x 2^{1-\sin x} + \frac{1}{\sqrt{1+e^x}}$ ,
20.  $f(x) = \ln(x + \sqrt{x^2+k})$
21.  $f(x) = \operatorname{arc} \sin \sqrt{\frac{1-x}{1+x}}$ ,
22.  $f(x) = \operatorname{arc} \operatorname{tg} \frac{1}{1+x}$ .

**Answers.** 1.  $f'(x) = 64x^3 + 24x^2 + 2x$ ; 2.  $f'(x) = \frac{2}{3} \frac{1+\sqrt[3]{x}}{(\sqrt[3]{x})^2}$ ; 3.  $f'(x) = -\frac{1}{2x^6}$ ; 4.  $f'(x) = -\frac{1}{(2x-1)^2}$ ; 5.  $f'(x) = \frac{1}{2} \frac{x-1}{(\sqrt{x})^3}$ ; 6.  $f'(x) = \frac{13x+5}{\sqrt{(2x+1)^2 + (3x+1)^2}}$ ; 7.  $f'(x) = 2x e^{x^2}$ ; 8.  $f'(x) = \cos x e^{\sin x}$ ; 9.  $f'(x) = \frac{\cos x}{2\sqrt{\sin x}} e^{\sqrt{\sin x}}$ ; 10.  $f'(x) = -\frac{\cos x}{\sin^2 x} e^{\frac{1}{\sin x}}$ ; 11.  $f'(x) = \cos \sqrt{1+x^2} \frac{x}{\sqrt{1+x^2}}$ ; 12.  $f'(x) = \frac{1+2x^2}{\sqrt{1+x^2}}$ ; 13.  $f'(x) = \operatorname{ctg}^2 \frac{x}{3} \frac{-1}{\sin^2 \frac{x}{3}}$ ; 14.  $f'(x) = 3 \sin^2 x \cos x - 3 \cos^2 x \sin x$ ; 15.  $f'(x) = 6x \sin^2 x^2 \cos x^2$ ; 16.  $f'(x) = \frac{1}{\sqrt{\operatorname{arc} \sin x}} \frac{1}{\sqrt{1-x^2}} - 3 (\operatorname{arc} \operatorname{tg} x)^2 \frac{1}{1+x^2}$ ; 17.  $f'(x) = \frac{-2}{x\sqrt{x^4-1}} + 5 \frac{\ln^4 x}{x}$ ; 18.  $f'(x) = -3^{-\sqrt{x}} \frac{1}{2\sqrt{x}} \ln 3 + \frac{-x^2+1}{(1+x^2)^2}$ ; 19.  $f'(x) = 2^{1-\sin x} - x 2^{1-\sin x} \cos x \ln 2 + \frac{-1}{2(\sqrt{1+e^x})^3} e^x$ ; 20.  $f'(x) = \frac{1}{\sqrt{x^2+k}}$ ; 21.  $f'(x) = -\frac{1}{2} \frac{\sqrt{2}}{\sqrt{x+x^2}\sqrt{1-x^2}}$ ; 22.  $f'(x) = \frac{1}{1+(1+x)^2} \frac{-1}{(1+x)^2}$ .

**Exercise 2** Find the second order derivatives of the following functions:

101.  $f(x) = \sin^2 x$ ,
102.  $f(x) = e^{-x^2}$ ,
103.  $f(x) = \ln(1+x^2)$ ,
104.  $f(x) = \operatorname{arc} \operatorname{tg} 2x$ .

**Answers.** 101.  $f'(x) = \sin 2x$ ,  $f''(x) = 2 \cos 2x$ ; 102.  $f'(x) = -2xe^{-x^2}$ ,  $f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$ ; 103.  $f'(x) = \frac{2x}{1+x^2}$ ,  $f''(x) = 2 \frac{1-x^2}{(1+x^2)^2}$ ; 104.  $f'(x) = \frac{2}{1+4x^2}$ ,  $f''(x) = -\frac{16x}{(1+4x^2)^2}$ .

**Exercise 3**

Compute the integrals:

1.  $\int \frac{\ln x}{x} dx$ ;
2.  $\int e^x \sin e^x dx$ ;
3.  $\int e^x \sqrt{e^x} dx$ ;
4.  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ ;
5.  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$ ;
6.  $\int \frac{dx}{x \ln x}$ ;
7.  $\int \frac{(\arcsin x)^3}{\sqrt{1-x^2}} dx$ ;
8.  $\int x \sqrt{3-x^2} dx$ ;
9.  $\int \frac{x}{\sqrt{3-x^2}} dx$ ;
10.  $\int \frac{\cos x dx}{\sqrt{\sin x}}$ ;
11.  $\int \operatorname{tg} x dx$ ;
12.  $\int (\operatorname{tg} x)^2 dx$ ;
13.  $\int \sqrt{\sin x} \cos x dx$ ;
14.  $\int (\sin x)^5 \cos x dx$ ;
15.  $\int (\cos x)^3 dx$ ;
16.  $\int x \cos x dx$ ;
17.  $\int x e^x dx$ ;
18.  $\int \ln x dx$ ;

Answers. **1.**  $\frac{1}{2} \ln^2 x + C$ ; **2.**  $-\cos(e^x) + C$ ; **3.**  $\frac{2}{3} e^{\frac{3}{2}x} + C$ ; **4.**  $2e^{\sqrt{x}} + C$ ;  
**5.**  $2 \sin \sqrt{x} + C$ ; **6.**  $\ln |\ln x| + C$ ; **7.**  $\frac{1}{4} (\arcsin x)^4 + C$ ; **8.**  $\frac{1}{3} (3-x^2)^{\frac{3}{2}} + C$ ;  
**9.**  $-\sqrt{3-x^2} + C$ ; **10.**  $2 \sin^{\frac{1}{2}} x + C$ ; **11.**  $-\ln |\cos x| + C$ ; **12.**  $\operatorname{tg} x - x + C$ ;  
**13.**  $\frac{2}{3} \sin^{\frac{3}{2}} x + C$ ; **14.**  $\frac{1}{6} (\sin x)^6 + C$ ; **15.**  $\sin x - \frac{1}{3} (\sin x)^3 + C$ ;  
**16.**  $\cos x + x \sin x + C$ ; **17.**  $x e^x - e^x + C$ ; **18.**  $x \ln x - x + C$ ;

### Exercise 4

Compute the integrals:

1.  $\int_0^{\frac{\pi}{2}} \sin x dx$ ,
2.  $\int_0^2 2^x dx$ ,
3.  $\int_1^6 \frac{dx}{1+\sqrt{3x-2}}$ ,  $t^2 = \sqrt{3x-2}$ ,
4.  $\int_1^e \ln x dx$ ,  $t = \ln x$ ,
5.  $\int_0^3 \sqrt{9-x^2} dx$ ,  $x = 3 \sin t$ ,
6.  $\int_0^1 x \sqrt{1+x} dx$ ,  $t = \sqrt{1+x}$ ,
7.  $\int_0^{\frac{1}{2} \ln 3} \frac{e^x dx}{1+e^{2x}}$ ,
8.  $\int_0^{\pi} \sin x e^{\cos x} dx$ ,
9.  $\int_0^{\frac{\pi}{4}} x \sin x dx$ .

Answers. **1.** 1; **2.**  $\frac{3}{\ln 2}$ ; **3.**  $2 - \frac{2}{3} \ln \frac{5}{2}$ ; **4.** 1; **5.**  $\frac{9\pi}{4}$ ; **6.**  $\frac{4}{15} (1 + \sqrt{2})$ ;  
**7.**  $\frac{\pi}{12}$ ; **8.**  $e - \frac{1}{e}$ ; **9.**  $\frac{4-\pi}{4\sqrt{2}}$ ;